International Journal of Theoretical Physics, Vol. 37, No. 7, 1998

Relativistic Covariant Equal-Time Equation for Quark–Diquark System

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Received January 7, 1998

Relativistic three-dimensional quasipotential (equal-time) equations are considered which describe bound states of a fermion and a boson of spin $S = 0$ or $S = 1$. The spin structure of the interaction quasipotentials in such systems is studied, and the corresponding partial-wave equation for the simplest case is obtained. Such equations can be used in calculations of energy spectra, decay rates, and structure functions of quark–diquark systems (nucleons and their resonances) and for the description of the ($\pi\ \mu$) atom as well.

1. INTRODUCTION

The concept of constituent diquarks was introduced in 1966.[(1)] In a three-quark system spin–spin interaction can lead to the existence of the short-range correlations in two-quark subsystems[(2)] which are comparable in strength to the $\bar{q}q$ attraction inside mesons. There is experimental evidence for diquark correlations in baryons.[(3),5] Scalar diquarks are mentioned in ref. 5 to be energetically favored. Moreover, in a series of recent papers[(6)] it was shown that the concept of diquarks as effective degrees of freedom arising

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[5] For another point of view see, e.g., refs. 4.

as a result of such correlations has important meaning for descriptions of nonleptonic weak decays at low energies. Scalar diquarks also arise in superstring-inspired models.[(7)] Therefore, the nucleon can be interpreted as the quark–diquark bound state and described on the basis of well-known methods for solving two-body problem. In connection with this we mention the papers of Lichtenberg[(8)] and Efimov's group.[(9),6] There are also many speculations concerning "diquonia" (diquark–antidiquark bound system) and "dibaryons," which are rather based on the radical point of view of considering diquarks as elementary constituents.

We also note that a new trend in the physics of elementary particles connected with research on the properties of so-called exotic atoms has been developed. Such systems represent atoms in which one of the electrons is replaced by an elementary particle.[(12)] The first work devoted to the consideration of these systems appeared in the forties:[(13)] In the mid 1970s the bound state of π-meson and muon,[(14)] which also can be interpreted as an exotic atom, was experimentally observed. The main properties of the $(\pi\ \mu)$ atom were studied theoretically in ref. 15 even before its experimental discovery. In these papers attention was paid to the possibilities of exploring the features of the π-meson by experimental investigation of the composite system of the meson and lepton (for subsequent work see ref. 16).

In ref. 17 the influence of relativistic effects in the description of the $(\pi\ \mu)$ atom was studied. For this purpose the equal-time quasipotential approach suggested by Logunov and Tavkhelidze[(18)] was used for the description of these bound states on the basis of quantum field methods.

In the present paper we employ the quasipotential method, in Kadyshevsky's version,[(19)] to the model in which the nucleon is considered to be a bound state of a quark of spin 1/2 and a diquark whose spin is $S = 0$ or $S = 1$. We are interested in the spin structure of the quasipotentials for interaction between a fermion (e.g., quark or μ-meson) and (pseudo) scalar particle (e.g., diquark or π-meson) as well as between a fermion and a vector particle which is described by the Joos–Weinberg formalism.[(20)] We also find the form of the quasipotential in the partial-wave equation for the $(\pi\mu)$ atom[7] and propose ways of numerical solution of the above-mentioned equations.

2. THE EQUATION FOR THE WAVE FUNCTION OF THE COMPOSITE SYSTEM FORMED BY A FERMION AND AN $S = 0$ BOSON

The quasipotential equation for the wave function of a composite system consisting of a fermion and a spinless boson has been obtained in ref. 22:

[6] See also the recent reviews in refs. 10 and 11.
[7] The analogous problem for the two spinor-particle system has been solved in ref. 21.

$$2\Delta^{0}_{p,m_2\lambda_P}\,(M - \Delta^{0}_{p,m_1\lambda_P} - \Delta^{0}_{p,m_2\lambda_P})\Phi_{\sigma}(\overset{\circ}{\vec{\Delta}}_{p,\lambda_P})$$

$$= \frac{1}{(2\pi)^3}\sum_{\nu}\int \frac{d^3\,\overset{\circ}{\vec{\Delta}}_{k,\lambda_P}}{2\Delta^{0}_{k,m_1\lambda_P}}\,V^{\nu}_{\sigma}\,(\overset{\circ}{\vec{\Delta}}_{p,\lambda_P};\,\overset{\circ}{\vec{\Delta}}_{k,\lambda_P})\,\Phi_{\nu}\,(\overset{\circ}{\vec{\Delta}}_{k,\lambda_P}) \qquad (2.1)$$

The quasipotential $\hat{V}$ coincides with the scattering amplitude of a muon on a pion in the first approximation in the coupling constant. The 4-momenta of particles covariantly defined in the c.m.s. are given by[(22–24),8] $[\lambda_P = (p_1 + p_2)/\sqrt{(p_1 + p_2)^2})]$

$$\overset{\circ}{\vec{\Delta}}_{p,\lambda_P} = \overset{\circ}{\vec{\Delta}}_{p,m_1\lambda_P} = (L^{-1}_{\lambda_P}\vec{p}_1) = \vec{p}_1 - \frac{\vec{P}}{M}\left(p^0_1 - \frac{\vec{P}\cdot\vec{p}_1}{P_0 + M}\right)$$

$$= -\overset{\circ}{\vec{\Delta}}_{p,m_2\lambda_P} \equiv \overset{\circ}{\vec{p}} \qquad (2.2)$$

$$\Delta^{0}_{p,m_j\lambda_P} = \sqrt{\overset{\circ}{\vec{\Delta}}{}^2_{p,\lambda_P} + m_j^2} \equiv \overset{\circ}{p}{}^{0}_{1}; \qquad j = 1, 2 \qquad (2.3)$$

Here M is the mass of bound system, m_1 is the muon mass, m_2 is the pion mass, $L^{-1}_{\lambda_P}$ is the matrix of the Lorentz boost from the system with 4-momentum P_μ and 4-velocity $\lambda^{\mu}_P \equiv P^{\mu}/\sqrt{P^2}$ to the rest system, and $L^{-1}_{\lambda_P}\,P = (M, \vec{0})$. The covariant 4-momentum of the particle after interaction ($\Delta^{0}_{k,m_j\lambda_P}$ and $\overset{\circ}{\vec{\Delta}}_{k,\lambda_P}$) is defined similarly.

In ref. 23 an expression for the quasipotential is given in the form

$$\hat{V}^{(2)\,\nu}_{\sigma}\,(\overset{\circ}{\vec{\Delta}}_{p,\lambda};\,\overset{\circ}{\vec{\Delta}}_{k,\lambda})$$

$$= \sum_{pol.inds.} D^{+(S=1/2)}_{\sigma\sigma_p}\{V^{-1}\,(\Lambda_P, p_1)\}$$

$$\times\, V_0\,(\overset{\circ}{\vec{\Delta}}_{k,\lambda} - \overset{\circ}{\vec{\Delta}}_{p,\lambda})\bar{u}(\overset{\circ}{\vec{\Delta}}_{p,\lambda};\,\sigma_p)\gamma_{\mu}u\,(\overset{\circ}{\vec{\Delta}}_{k,\lambda};\,\nu_p)\,(\tilde{\Delta}_{p,\lambda} + \tilde{\Delta}_{k,\lambda})^{\mu}$$

$$\times\, D^{(S=1/2)}_{\nu_p\nu_k}\{V^{-1}(\Lambda_{p_1}, k_1)\}\,D^{(S=1/2)}_{\nu_k\nu}\,\{V^{-1}(\Lambda_P, k_1)\} \qquad (2.4)$$

where V_0 is the local part of the quasipotential corresponding to the one-boson exchange, $\Delta^{\mu}_{p,\lambda_P} = (\Delta^{0}_{p,m_1\lambda_P};\,\overset{\circ}{\vec{\Delta}}_{p,\lambda_P})$; $\tilde{\Delta}^{\mu}_{p,\lambda_P} = (\Delta^{0}_{p,m_2\lambda_P};\,-\overset{\circ}{\vec{\Delta}}_{p,\lambda_P})$; $D^{(S=1/2)}$ are the Wigner functions.

Let us rewrite (2.4) in detail using the results of ref. 24. We employ the expression of ref. 24 for the 4-current

$$j^{\mu}_{\sigma_p\nu_p}(\vec{p}, \vec{k}) = \bar{u}(\vec{p}, \sigma_p)\gamma^{\mu}u\,(\vec{k}, \nu_p)$$

$$= \frac{2}{\sqrt{2m(\Delta^0 + m)}}\,\xi^{\dagger}_{\sigma_p}[\,p^{\mu}(\Delta_0 + m) + 2W^{\mu}(\vec{p})(\vec{\sigma}\cdot\vec{\Delta})]\,\xi_{\nu_p} \qquad (2.5)$$

[8] We omit the open dots above $\vec{p}$ and $\vec{k}$ in the following, implying still the covariant generalizations of the usual momenta.

where

$$\vec{\Delta} \equiv \vec{k}(-)\vec{p} = (L_{\vec{p}}^{-1}\vec{k}); \qquad \Delta^0 \equiv \sqrt{\vec{\Delta}^2 + m^2} = \frac{p^\mu k_\mu}{m} \tag{2.6}$$

is the momentum transfer in the Lobachevsky space. $W^\mu(\vec{p})$ is the vector of relativistic spin [Pauli–Lubanski–Shirokov vector, $p_\mu W^\mu(\vec{p}) = 0$; $k_\mu W^\mu(\vec{p}) = -(m/2)(\vec{\sigma}\vec{\Delta})$]. We "reset" the polarization indices to a single momentum, e.g., $\vec{p}$ as earlier.[(21,24–26)] As a result we obtain[9]

$$\begin{aligned}\hat{V}^{(2)}(\vec{k}, \vec{p}) = \frac{2}{\sqrt{2m_1(\Delta_1^0 + m_1)}} \{ &[p_1^0(p_1^0 + p_2^0 + k_1^0 + k_2^0) - 2m_1^2](\Delta_1^0 + m_1) \\ &+ (\vec{p} \cdot \vec{\Delta}_1)(p_1^0 + p_2^0 + k_1^0 + k_2^0) \\ &+ i\vec{\sigma} \cdot [\vec{p} \times \vec{\Delta}_1](p_1^0 + p_2^0 + k_1^0 + k_2^0)\} \, V_0(\vec{k}(-)\vec{p})\end{aligned} \tag{2.7}$$

After transition to the nonrelativistic limit[10] one can see that the quasipotential (2.7) transforms to the following form ($\vec{\Delta}_{\mathscr{E}} = \vec{k} - \vec{p}$):

$$\begin{aligned}V^{(2)}_{nonrel}(\vec{k}, \vec{p}) = &-g_V^2 \frac{4m_1m_2}{\vec{\Delta}^2_{\mathscr{E}}} + g_V^2 \frac{1}{c^2}\left(1 + \frac{m_2}{2m_1}\right) \\ &- g_V^2 \frac{1}{c^2}\left(2 + \frac{m_1}{m_2} + \frac{m_2}{m_1}\right)\frac{\vec{p}^2 + \vec{k}^2}{\vec{\Delta}^2_{\mathscr{E}}} \\ &- g_V^2 \frac{1}{c^2}\frac{m_2}{m_1}\frac{(\vec{k}^2 - \vec{p}^2)^2}{\vec{\Delta}^4_{\mathscr{E}}} - g_V^2\frac{1}{c^2}\left(1 + \frac{m_2}{m_1}\right)\frac{2i\vec{\sigma} \cdot [\vec{p} \times \vec{\Delta}_{\mathscr{E}}]}{\vec{\Delta}^2_{\mathscr{E}}}\end{aligned} \tag{2.8}$$

provided that the local part of the quasipotential is chosen in the form

$$V_0(\vec{k}(-)\vec{p}) = \frac{g_V^2}{(p_1 - k_1)^2} = -\frac{g_V^2}{2m_1(\Delta_1^0 - m_1)} \tag{2.9}$$

g_V is the coupling constant for the quark–vector boson and diquark–vector boson interactions. After some calculations we obtain the matrix elements of the quasipotential (2.7), $\hat{V}^{\nu}_{\sigma}(\vec{k}, \vec{p})$. They can be written in the following form:

[9] Another version of the quasipotential approach, based on the two-time Green function formalism, has been used in ref. 27. This approach leads to the quasipotentials depending explicitly on the total energy of the bound system.

[10] More exactly, to the quasirelativistic limit, similar to the use of the $1/c^2$ expansion in the Breit equation for the two-spinor-particle interaction.[(28)]

$$V_{1/2}^{-1/2} = \frac{2V_0(\bar{k}(-)\bar{p})}{\sqrt{2m_1(\Delta_1^0 + m_1)}}(p_1^0 + p_2^0 + k_1^0 + k_2^0)(i\eta_1 + \eta_2) \tag{2.10}$$

$$V_{-1/2}^{1/2} = \frac{2V_0(\bar{k}(-)\bar{p})}{\sqrt{2m_1(\Delta_1^0 + m_1)}}(p_1^0 + p_2^0 + k_1^0 + k_2^0)(i\eta_1 - \eta_2) \tag{2.11}$$

$$V_{1/2}^{1/2} = \frac{2V_0(\bar{k}(-)\bar{p})}{\sqrt{2m_1(\Delta_1^0 + m_1)}}[(p_1^0(p_1^0 + p_2^0 + k_1^0 + k_2^0) - 2m_1^2)(\Delta_1^0 + m_1) + (p_1^0 + p_2^0 + k_1^0 + k_2^0)(\bar{p}\cdot\bar{\Delta}_1) + i\eta_3(p_1^0 + p_2^0 + k_1^0 + k_2^0)] \tag{2.12}$$

$$V_{-1/2}^{-1/2} = \frac{2V_0(\bar{k}(-)\bar{p})}{\sqrt{2m_1(\Delta_1^0 + m_1)}}[(p_1^0(p_1^0 + p_2^0 + k_1^0 + k_2^0) - 2m_1^2)(\Delta_1^0 + m_1) + (p_1^0 + p_2^0 + k_1^0 + k_2^0)(\bar{p}\cdot\bar{\Delta}_1) - i\eta_3(p_1^0 + p_2^0 + k_1^0 + k_2^0)] \tag{2.13}$$

where $\bar{\eta} = [\bar{p} \times \bar{\Delta}]$.

Expanding the wave function and the quasipotential in partial waves, we obtain the system of partial equations[26]

$$2p_2^0(M - p_1^0 - p_2^0)\frac{1}{p}\Psi_{Jl}(p) = \frac{1}{2\pi}\int_0^\infty \frac{k\,dk}{k_1^0}\sum_{l'} V_{ll'}^J(k, p)\Psi_{Jl'}(k) \tag{2.14}$$

where $J = |l - 1/2|$, $l + 1/2$, $k = |\bar{\Delta}_{k,\lambda}|$; $p = |\bar{\Delta}_{p,\lambda}|$. The coefficients $V_{ll'}^J(k, p)$ can be found by the formula

$$V_{ll'}^J(k, p) = \sum_{M,\sigma,\nu}\int_0^\pi \sin\theta_p\, d\theta_p \int_0^{2\pi} d\phi_p \int_0^\pi \sin\theta_k\, d\theta_k \int_0^{2\pi} d\phi_k \times [\Omega_{JlM}^{*(1/2)}(\bar{n}_p)]^\sigma V_\sigma^\nu(\bar{k}(-)\bar{p};\bar{p})\,[\Omega_{JlM}^{1/2}(\bar{n}_k)]_\nu \tag{2.15}$$

Here, $\bar{n}_p = \bar{p}/|\bar{p}|$, $\bar{n}_k = \bar{k}/|k|$, θ_p, ϕ_p are angular coordinates of the vector $\bar{n}_p$; θ_k, ϕ_k are angular coordinates of the vector $\bar{n}_k$; $\Omega_{JlM}^{(1/2)}(\bar{n})$ are spherical spinors.

Let us choose the coordinate system in such a way that the vector $\bar{n}_p$ is aligned to the OZ axis and the vector $\bar{n}_k$ lies in the XZ plane. The results of calculations can be expressed in terms of the integrals $I_1^{(l)}$ and $I_2^{(l)}$:

$$V_{ll}^{l\pm1/2}(p,k) = -\left(J + \frac{1}{2}\right)\frac{g_V^2}{m_1\sqrt{2pk}}\left\{\left[p_1^0(p_1^0 + p_2^0 + k_1^0 + k_2^0)\gamma^+ - 2m_1^2(\gamma^+ - \gamma^-) - \frac{p}{k}k_1^0(p_1^0 + p_2^0 + k_1^0 + k_2^0)\right]I_1^{(l)} - 2m_1^2 I_2^{(l)}\right\} \mp \left(J + \frac{1}{2}\right)\frac{g_V^2}{\sqrt{2pk}}\left(p_1^0 + p_2^0 + k_1^0 + k_2^0\right)\frac{l(l+1)}{2l+1}$$

$$\times (I_1^{(l-1)} - I_1^{(l+1)})(1 - \delta_{l0}) \tag{2.16}$$

The matrix element for the $\Delta l = \pm 1$ transition drops out,

$$V^{l\pm 1/2}_{l,l\pm 1}(p, k) = 0 \tag{2.17}$$

Here, $\gamma^+ = (p_1^0 k_1^0 + m_1^2)/pk$, $\gamma^- = (p_1^0 k_1^0 - m_1^2)/pk$, and

$$I_1^{(l)} = \int_{-1}^{1} dz \frac{P_l(z)}{(\gamma^- - z)\sqrt{\gamma^+ - z}} \tag{2.18}$$

$$I_2^{(l)} = \int_{-1}^{1} dz \frac{P_l(z)}{\sqrt{\gamma^+ - z}} \tag{2.19}$$

$P_l(z)$ is the Legendre polynomial of the first kind.

The value of the second integral can be taken from ref. 29, p. 822,

$$I_2^{(l)} = \frac{2^{1-l}}{2\,l + 1}\left(\sqrt{\gamma^+ + 1} - \sqrt{\gamma^+ - 1}\right)^{2l+1} \tag{2.20}$$

In cases of low angular momenta ($l = 0, 1, 2$), the first integral can be directly calculated from (2.18),

$$I_1^{(0)} = \frac{1}{\sqrt{\gamma^+ - \gamma^-}} \times \ln\left[\frac{(\sqrt{\gamma^+ + 1} - \sqrt{\gamma^+ - \gamma^-})(\sqrt{\gamma^+ - 1} + \sqrt{\gamma^+ - \gamma^-})}{(\sqrt{\gamma^+ + 1} + \sqrt{\gamma^+ - \gamma^-})(\sqrt{\gamma^+ - 1} - \sqrt{\gamma^+ - \gamma^-})}\right] \tag{2.21}$$

$$I_1^{(1)} = \gamma^- I_1^{(0)} - 2\left(\sqrt{\gamma^+ + 1} - \sqrt{\gamma^+ - 1}\right) \tag{2.22}$$

$$I_1^{(2)} = \left(\frac{3}{2}\gamma^{-2} - \frac{1}{2}\right) I_1^{(0)} - \left(\gamma^+ - \sqrt{\gamma^{+2} - 1} + 3\gamma^-\right) \times \left(\sqrt{\gamma^+ + 1} - \sqrt{\gamma^+ - 1}\right) \tag{2.23}$$

However, calculation of the first integral for arbitrary l, the orbital quantum number, is highly complicated and the result seems not to be expressed in terms of known special functions. See the Appendix for some speculations in connection with this subject.

3. THE EQUATION FOR THE WAVE FUNCTION OF THE COMPOSITE SYSTEM FORMED BY A FERMION AND A $S = 1$ BOSON

The equation for the equal-time WF of the composite system of a fermion and a $S = 1$ boson is analogous to the one presented in Section 2,

$$2\Delta^0_{p,m_2\lambda_P}(M - \Delta^0_{p,m_1\lambda_P} - \Delta^0_{p,m_2\lambda_P})\Phi_{\sigma_1\sigma_2}(\overline{\Delta}_{p,\lambda_P})$$
$$= \frac{1}{(2\pi)^3}\sum_{\nu_1\nu_2}\int \frac{d^3\overline{\Delta}_{k,\lambda_P}}{2\Delta^0_{k,m_1\lambda_P}} V^{\nu_1\nu_2}_{\sigma_1\sigma_2}(\overline{\Delta}_{p,\lambda_P};\overline{\Delta}_{k,\lambda_P})\Phi_{\nu_1\nu_2}(\overline{\Delta}_{k,\lambda_P}) \quad (3.24)$$

It is obvious that in this case the quasipotential in the momentum representation has additional terms (which are responsible for the spin–spin interaction, the tensor interaction, and the squared spin–orbit interaction) compared to the case of the fermion–$S = 0$ boson.[30]

Following the technique of "resetting" the polarization indices, we get, analogous to Section 2,[11]

$$\langle p_1, p_2; \sigma_1, \sigma_2 | \hat{V}^{(2)} | k_1, k_2; \nu_1, \nu_2\rangle$$
$$= \sum_{\text{pol.inds.}} D^{+(S=1/2)}_{\sigma_1\sigma_{1p}}\{V^{-1}(\Lambda_P, p_1)\}\, D^{+(S=1)}_{\sigma_2\sigma_{2p}}\{V^{-1}(\Lambda_P, p_2)\}$$
$$\times V^{\nu_{1p}\nu_{2p}}_{\sigma_{1p}\sigma_{2p}}(\overline{k}(-)\overline{p}, \overline{p})\, D^{(S=1/2)}_{\nu_{1p}\nu_{1k}}\{V^{-1}(\Lambda_{p_1}, k_1)\}\, D^{(S=1/2)}_{\nu_{1k}\nu_1}\{V^{-1}(\Lambda_P, k_1)\}$$
$$\times D^{(S=1)}_{\nu_{2p}\nu_{2k}}\{V^{-1}(\Lambda_{p_2}, k_2)\} D^{(S=1)}_{\nu_{2k}\nu_2}\{V^{-1}(\Lambda_P, k_2)\} \quad (3.25)$$

$$V^{\nu_{1p}\nu_{2p}}_{\sigma_{1p}\sigma_{2p}}(\overline{k}(-)\overline{p}, \overline{p}) = \xi^{\dagger}_{\sigma_{1p}}\xi^{\dagger}_{\sigma_{2p}}\hat{V}^{(2)}(\overline{k}(-)\overline{p}, \overline{p})\xi_{\nu_{1p}}\xi_{\nu_{2p}} \quad (3.26)$$

Let us use the equations for the 4-current of a spinor particle defined by formula (2.5) and (3.27) for the 4-current of a vector particle in the Joos–Weinberg formalism,

$$j^{\mu}_{\sigma_{2p}\nu_{2p}}(\overline{p}, \overline{k})$$
$$= \xi^{\dagger}_{\sigma_{2p}}\left[(p_2 + k_2)^{\mu} + \frac{1}{m_2}W^{\mu}(\overline{p}_2)(\overline{S}_2\overline{\Delta}_2) - \frac{1}{m_2}(\overline{S}_2\overline{\Delta}_2)W^{\mu}(\overline{p}_2)\right]\xi_{\nu_{2p}} \quad (3.27)$$

Following the rules of construction of the quasipotential over the on-shell scattering amplitude,[18,19] we obtain

$$\langle \overline{p}_1, \overline{p}_2; \sigma_{1p}, \sigma_{2p} | \hat{V}^{(2)} | \overline{k}_1, \overline{k}_2; \nu_{1p}, \nu_{2p}\rangle$$
$$= \langle \overline{p}_1, \overline{p}_2; \sigma_{1p}, \sigma_{2p} | \hat{T}^{(2)} | \overline{k}_1, \overline{k}_2; \nu_{1p}, \nu_{2p}\rangle$$
$$= -g_V^2 \frac{j^{\mu}_{\sigma_{1p}\nu_{1p}}(\overline{p}_1, \overline{k}_1)\, g_{\mu\nu} j^{\nu}_{\sigma_{2p}\nu_{2p}}(\overline{p}_2, \overline{k}_2)}{(p_1 - k_1)^2} \quad (3.28)$$

where, as earlier, $\overline{p} = \overline{p}_1 = -\overline{p}_2$ and $\overline{k} = \overline{k}_1 = -\overline{k}_2$ are the covariant generalizations of momenta.

[11] Note that $\xi_{\sigma_{1p}}, \xi_{\nu_{1p}}$ are the usual Pauli two-component spinors normalized by the equation $\xi^{\dagger}_{\sigma}\xi^{\nu} = \delta^{\nu}_{\sigma}$, and $\xi_{\sigma_{2p}}, \xi_{\nu_{2p}}$ are 3-component analogues of the Pauli spinors for the $S = 1$ particle.

As a result one can write the quasipotential operator as follows:

$$
\begin{aligned}
\hat{V}^{(2)}(\overline{p},\overline{k}) = &- g_V^2 \sqrt{\frac{\Delta_1^0 + m_1}{2m_1}}\, \frac{p_1^0 p_1^0 + p_2^0 + k_1^0 + k^0{}_2) - 2m_1^2}{m_1(\Delta_1^0 - m_1)} \\
&- g_V^2 \frac{(p_1^0 + p_2^0 + k_1^0 + k_2^0)(\overline{p}\cdot\overline{\Delta}_1)}{m_1(\Delta_1^0 - m_1)\sqrt{2m_1(\Delta_1^0 + m_1)}} \\
&- g_V^2 \frac{i\overline{\sigma}_1\cdot[\overline{p}\times\overline{\Delta}_1](p_1^0 + p_2^0 + k_1^0 + k_2^0)}{m_1(\Delta_1^0 - m_1)\sqrt{2m_1(\Delta_1^0 + m_1)}} \\
&+ g_V^2 \sqrt{\frac{\Delta_1^0 + m_1}{2m_1}}\, \frac{i\overline{S}_2\cdot[\overline{p}\times\overline{\Delta}_2]\,(p_1^0 + p_2^0)}{m_1 m_2(\Delta_1^0 - m_1)} \\
&+ g_V^2 \frac{1}{\sqrt{2m_1(\Delta_1^0 + m_1)}} \\
&\times \frac{i\overline{S}_2\cdot[\overline{p}\times\overline{\Delta}_2](\overline{p}\cdot\overline{\Delta}_1)\,(p_1^0 + p_2^0 + m_1 + m_2)^2}{2m_1 m_2(\Delta_1^0 - m_1)(p_1^0 + m_1)(p_2^0 + m_2)} \\
&+ g_V^2 \sqrt{\frac{m_1}{2(\Delta_1^0 + m_1)}} \\
&\times \frac{(\overline{\sigma}_1\cdot\overline{\Delta}_2)(\overline{S}_2\cdot\overline{\Delta}_1) - (\overline{\sigma}_1\cdot\overline{S}_2)(\overline{\Delta}_1\cdot\overline{\Delta}_2) + i\overline{S}_2\cdot[\overline{\Delta}_1\times\overline{\Delta}_2]}{m_1(\Delta_1^0 - m_1)} \\
&- g_V^2 \frac{1}{\sqrt{2m_1(\Delta_1^0 + m_1)}} \\
&\times \frac{\overline{\sigma}_1\cdot[\overline{p}\times\overline{\Delta}_1]\overline{S}_2\cdot[\overline{p}\times\overline{\Delta}_2](p_1^0 + p_2^0 + m_1 + m_2)^2}{m_1 m_2(\Delta_1^0 - m_1)(p_1^0 + m_1)(p_2^0 + m_2)}
\end{aligned}
\tag{3.29}
$$

Here,

$$
\begin{aligned}
\overline{\Delta}_1 &= \overline{k} - \frac{\overline{p}}{m_1}\left(k_1^0 - \frac{\overline{k}\cdot\overline{p}}{p_1^0 + m_1}\right), \qquad \Delta_1^0 = \sqrt{\overline{\Delta}_1^2 + m_1^2} \\
\overline{\Delta}_2 &= \overline{k} - \frac{\overline{p}}{m_2}\left(k_2^0 - \frac{\overline{k}\cdot\overline{p}}{p_2^0 + m_2}\right), \qquad \Delta_2^0 = \sqrt{\overline{\Delta}_2^2 + m_2^2}
\end{aligned}
\tag{3.30}
$$

and $p_1^0 = \sqrt{\overline{p}^2 + m_1^2}$, $k_1^0 = \sqrt{\overline{k}^2 + m_1^2}$, $p_2^0 = \sqrt{\overline{p}^2 + m_2^2}$, and $k_2^0 = \sqrt{\overline{k}^2 + m_2^2}$.

In the quasirelativistic approximation (taking account of terms up to the order $1/c^2$), equation (3.29) yields

$$
\begin{aligned}
V^{(2)}_{\mathrm{nonrel}}(\bar{k},\bar{p}) = {} & -g_V^2\frac{4m_1m_2}{\bar{\Delta}_\varepsilon^2} + g_V^2\frac{1}{c^2}\left(1+\frac{m_2}{2m_1}\right) \\
& - g_V^2\frac{1}{c^2}\left(2+\frac{m_1}{m_2}+\frac{m_2}{m_1}\right)\frac{\bar{p}^2+\bar{k}^2}{\bar{\Delta}_\varepsilon^2} \\
& - g_V^2\frac{1}{c^2}\frac{m_2}{m_1}\frac{(\bar{k}^2-\bar{p}^2)^2}{\bar{\Delta}_\varepsilon^4} - g_V^2\frac{1}{c^2}\left(1+\frac{m_2}{m_1}\right)\frac{2i\bar{\sigma}_1\cdot[\bar{p}\times\bar{\Delta}_\varepsilon^2]}{\bar{\Delta}_\varepsilon^2} \\
& - g_V^2\frac{2}{c^2}\left(1+\frac{m_1}{m_2}\right)\frac{i\bar{S}_2\cdot[\bar{p}\times\bar{\Delta}\varepsilon]}{\bar{\Delta}_\varepsilon^2} \\
& - g_V^2\frac{1}{c^2}\frac{(\bar{\sigma}_1\cdot\bar{\Delta}_\varepsilon)(\bar{S}_2\cdot\bar{\Delta}_\varepsilon)-(\bar{\sigma}_1\cdot\bar{S}_2)\bar{\Delta}_\varepsilon^2}{\bar{\Delta}_\varepsilon^2}
\end{aligned}
\tag{3.31}
$$

where again $\bar{\Delta}_\varepsilon = \bar{k} - \bar{p}$ is the momentum transfer in Euclidean space. Compared to (2.8) we have two additional terms corresponding to the tensor forces and the spin–orbit interaction of the second particle.

One can see that this case is more complicated than the case of Section 2 and does not admit analytical solution. Therefore, we intend to solve the equation numerically with the quasipotential (3.29) in the subsequent publications. Good accuracy in attempts at the numerical solution of such types of problems is provided by the spline method[31] or by the method for solving the spectral problems developed in ref. 32 based on Galerkin's procedure for the discretization of integral operators. They have been used for the description of a two-spinor system in ref. 33.

4. CONCLUSIONS

We have applied the covariant three-dimensional quasipotential approach to the description of quark–diquark bound states, which can be interpreted as nucleons and their resonances. We have derived the partial relativistic equal-time equation for the $(\pi\mu)$ atom and other bound systems (e.g., proton) composed of the particles with spin 1/2 (quark) and spin 0 (diquark). The spin structure of the quasipotential for the system of a fermion and an $S = 1$ boson also has been studied.

The presence of huge terms in these equations induces us to employ numerical methods for their solution.

ACKNOWLEDGMENTS

We would like to express our sincere gratitude to V. G. Kadyshevsky, R. N. Faustov, M. Moreno, N. B. Skachkov, Yu. N. Tyukhtyaev, and C. Villareal for interest in this work and for most helpful discussions. We greatly appreciate the technical assistance of A. S. Rodin.

One of us (V.D.) is very grateful to Prof. A. M. Cetto, Head of the Departamento de Física Teórica at the IFUNAM, for the creation of excellent conditions for work.

This work has been financially supported by CONACYT (Mexico) under contract No. 920193 and the Scientific and Technological Center for Control and Use of Physical Fields and Radiations (Saratov).

APPENDIX

The integral (2.18)

$$I_1^{(l)} = \int_{-1}^{1} \frac{P_l(z)\, dz}{(\gamma^- - z)\sqrt{\gamma^+ - z}} \tag{A.1}$$

can be reduced by means of simple algebraic transformations to

$$I_1^{(l)} = \frac{1}{\gamma^- - \gamma^+} I_2^{(l)} + \frac{2}{\sqrt{\gamma^+ - \gamma^-}} Q_l(\gamma^-) + \frac{1}{\gamma^+ - \gamma^-} \int_{-1}^{1} \frac{P_l(z)\, dz}{\sqrt{\gamma^+ - z} + \sqrt{\gamma^+ - \gamma^-}} \tag{A.2}$$

where $Q_l(x)$ is the Legendre function of the second kind. However, the calculation of the integral in (A.2) is as complicated as that of (A.1).

We can use the multiple Mellin transform and the tables of formulas from refs. 34 and 35 in order to calculate the integral (2.18). The multiple Mellin transform has the form

$$K^*(s_1, \ldots, s_n) = \int_0^\infty \cdots \int_0^\infty K(x_1, \ldots, x_n) x_1^{s_1 - 1} \ldots x_n^{s_n - 1}\, dx_1 \ldots dx_n \tag{A.3}$$

If the function $K(c_1, \ldots, c_n)$ can be represented in such a form

$$K(c_1, \ldots, c_n) = \int_0^\infty K_1(x) K_2\left(\frac{c_1}{x}\right) \ldots K_{n+1}\left(\frac{c_n}{x}\right) \frac{dx}{x} \tag{A.4}$$

then the transform $K(s_1, \dots, s_n)$ is calculated by the formula

$$K^*(s_1, \dots, s_n) = K_1^*(s_1 + \dots + s_n)K_2^*(s_1) \dots K_{n+1}^*(s_n) \qquad \text{(A.5)}$$

After the substitution $z = 2/x - 1$, the needed integral is rewritten as follows:

$$I_1^{(l)} = \frac{(-1)^l\sqrt{\gamma^+ - 1}}{\gamma^- - 1} \int_0^\infty \frac{dx}{x} P_l\left(\frac{2}{x} - 1\right)H(x - 1) \times \frac{c/x}{\sqrt{1 + c/x}\,\{1 + [(\gamma^+ - 1)/(\gamma^- - 1)]c/x\}} \qquad \text{(A.6)}$$

where $c = 2/(\gamma^+ - 1)$, and

$$H(y) = \begin{cases} 1, & y \geq 0 \\ 0, & y < 0 \end{cases}$$

is the Heaviside function.

Taking the transforms from the tables of refs. 35 and 36, we can use the inverse Mellin transformation to find the value of the integral of (A.6)

$$K(c_1, \dots, c_n) = \frac{1}{(2\pi i)^n} \int_{\gamma_1 - i\infty}^{\gamma_1 + i\infty} \dots \int_{\gamma_n - i\infty}^{\gamma_n + i\infty} K^*(s_1, \dots, s_n) c_1^{-s_1} \dots c_n^{-s_n}\, ds_1 \dots ds_n \qquad \text{(A.7)}$$

where $\gamma_k = \Re e\, s_k$, $k = 1, \dots, n$.

Thus,

$$I_1^{(l)} = (-1)^l \frac{\sqrt{\gamma^+ - 1}}{\gamma^- - 1} \sum_{k=0}^{\infty} \frac{1}{\Gamma\left(\frac{3}{2} + k\right)} \left(\frac{\gamma^- - \gamma^+}{\gamma^- - 1}\right)^k \times G_{3,3}^{1,3}\left(\frac{2}{\gamma^+ - 1}\,\middle|\, \begin{matrix} 1, & 1, & \frac{1}{2} \\ 1 + k, & -l, & 1 + l \end{matrix}\right) \qquad \text{(A.8)}$$

where $G_{B+C,A+D}^{A,B}$ is the Meijer G-function.

Another way is also possible: to consider every term in the integral

(A.1) separately and to employ triple[12] Mellin transforms. Using this technique,[(34)] we obtain

$$K_1(x) = P_l\left(\frac{2}{x} - 1\right)H(x-1) \Rightarrow K_1^*(s) = \Gamma\begin{bmatrix} -s, -s \\ l+1-s, -l-s \end{bmatrix} \quad \text{(A.9)}$$

$$K_2\left(\frac{c_1}{x}\right) = \frac{1}{x/c_1 - 1} \Rightarrow K_2^*(s_1) = -\pi\Gamma\begin{bmatrix} -s_1, 1+s_1 \\ \frac{1}{2} - s_1, \frac{1}{2} + s_1 \end{bmatrix} \quad \text{(A.10)}$$

$$K_3\left(\frac{c_2}{x}\right) = \frac{1}{\sqrt{1 - c_1/x}} \Rightarrow K_3^*(s_2) = \frac{\pi}{\Gamma(1/2)\cos(\pi/4)}\Gamma\begin{bmatrix} s_2, \frac{1}{2} - s_2 \\ \frac{1}{4} + s_2, \frac{3}{4} - s_1 \end{bmatrix} \quad \text{(A.11)}$$

where $c_1 = 2/(\gamma^- + 1)$, $c_2 = 2/(\gamma^+ + 1)$, and $s = s_1 + s_2$, $\Gamma(s)$ is the Eiler Γ-function, and $\Gamma\begin{bmatrix} a_1 \dots a_k \\ b_1 \dots b_m \end{bmatrix}$ denotes

$$\Gamma\begin{bmatrix} a_1 \dots a_k \\ b_1 \dots b_m \end{bmatrix} = \frac{\Gamma(a_1)\dots\Gamma(a_k)}{\Gamma(b_1)\dots\Gamma(b_m)}$$

Then,

$$K^*(s_1, s_2) = -\frac{\sqrt{2}\pi^2}{\Gamma(1/2)}$$

$$\Gamma\begin{bmatrix} -s_1 - s_2, -s_1 - s_2, -s_1, 1+s_1, s_2, \frac{1}{2} - s_2 \\ l+1-s_1-s_2, -l-s_1-s_2, \frac{1}{2} - s_1, \frac{1}{2} + s_1, \frac{1}{4} + s_2, \frac{3}{4} - s_2 \end{bmatrix} \quad \text{(A.12)}$$

Similar to the above calculation, employing the inverse Mellin transformation (A.7),

$$K(c_1, c_2) = \frac{1}{(2\pi i)^2}\int_{\gamma_1 - i\infty}^{\gamma_1 + i\infty}\int_{\gamma_2 - i\infty}^{\gamma_2 + i\infty} K^*(s_1, s_2)c_1^{-s_1}c_2^{-s_2}\, ds_1\, ds_2 \quad \text{(A.13)}$$

to our integral, we come to

$$K(c_1, c_2) = -\frac{\sqrt{2}\pi^2}{\Gamma(1/2)}\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{k+n}}{k!n!}$$

$$\times\Gamma\begin{bmatrix} k+n+1, k+n+1, k+1, \frac{1}{2}+n \\ l+k+n+2, -l+k+n+1, \frac{3}{2}+k, -\frac{1}{2}-k, \frac{1}{4}-n, \frac{3}{4}+n \end{bmatrix}$$
$$\times c_1^{k+1}c_2^{n} \quad \text{(A.14)}$$

[12]The number of multiple terms in the integral (A.1) is three.

which can be slightly simplified after the use of well-known expressions for the Γ-function, $\Gamma(p)\Gamma(1-p) = \pi/\sin p\pi$ and $\Gamma(k+1) = k!$ $(k = 0, 1, \ldots)$

Finally, the value of the integral (A.1) can be represented in the form of the complicated double sum of Γ-functions:

$$I_1^{(l)} = \frac{1}{\sqrt{\pi(\gamma^+ + 1)}} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty}$$

$$\Gamma\left[\begin{matrix} k+n+1, k+n+1, \frac{1}{2}+n \\ l+k+n+2, -l+k+n+1, n+1 \end{matrix}\right] c_1^{k+1} c_2^{n} \quad \text{(A.15)}$$

It is not clear which representation of the integral under consideration is more convenient; all of them are rather inconvenient for some applications. In our opinion, further simplifications appear to be impossible, and the use of a computer seems necessary.

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